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Beta Stable Neutron Stars in Non-Linear Relativistic Mean Field Models*

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Abstract

Properties of beta stable neutron stars are investigated in relativistic non-linear mean field models that correctly describe known bulk properties of symmetric nuclear matter. The requirement of beta stability can significantly affect the neutron matter equation of state and in turn the static properties of neutron stars.

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A number of calculations predicting the static properties of cold neutron stars have been done using a variety of neutron matter equations of state.[1] Neutron matter density at the center of the neutron star is believed to reach several times nuclear densities encountered in normal nuclei. To compute the properties of neutron stars using the Tolman-Oppenheimer-Volkov equations the neutron matter equation of state must be known at these large densities. This equation of state is usually calculated in models that violate the most elemental requirements, such as relativity. An exception to this criticism is the work of Chin and Walecka,[2] who used a particular relativistic field theory to obtain the equation of state at high densities.

The idea of a neutron star was originally conceived by Landau[3] immediately after learning of neutron's discovery. He imagined a heavenly body made of neutrons and held together by gravity. Strictly speaking, this idea cannot be correct, because of neutron's beta decay. A neutron star must have a certain percentage of protons and electrons. The requirement of beta stability in neutron stars has been neglected in the belief that its fulfillment would lead to a small and negligible proton concentration and only trivial modification of neutron matter equation of state. That this assumption can be incorrect for proton concentration in beta stable neutron matter was shown by the author.[4] It was shown that about twice nuclear density [$k_F(n) \approx 400$ MeV] beta stable neutron matter can be 15% protons. At neutron star densities, this percentage can be much larger. The presence of a large percentage of protons in neutron matter (electrons will keep the system electrically neutral) will affect the equation of state. The equation of state of beta stable neutron matter and the corresponding modifications of the static properties of neutron stars has never been investigated in detail. We find that in some models, the modification of the equation of state due to beta stability is substantial.

A useful method to study the properties of dense nuclear matter is relativistic quantum field theory in the mean field approximation. The Lagrangian describing the nucleons ψ interacting with isoscalar field σ, isoscalar vector field ω_μ and isovector field $\hat{R}_\mu$ is taken to be

$$\begin{aligned}\mathcal{L} = & -\bar{\psi}(\gamma_\mu \partial_\mu + m_N)\psi - \frac{1}{2}(\partial_\mu \sigma)^2 - U(\sigma) \\ & - \frac{1}{4} F_{\mu\nu} F_{\mu\nu} - \frac{1}{4} \vec{G}_{\mu\nu} \cdot \vec{G}_{\mu\nu} - \frac{1}{2} m_V^2 \omega_\lambda \omega_\lambda - \frac{1}{2} m_V^2 \hat{R}_\lambda \cdot \hat{R}_\lambda \\ & + i g_V \bar{\psi} \gamma_\lambda \psi \omega_\lambda + i g_r \bar{\psi} \gamma_\mu \vec{\tau} \cdot \psi \hat{R}_\mu - g_S \bar{\psi}\psi\sigma \end{aligned} \tag{1}$$

where

$$\begin{aligned} F_{\mu\nu} &= \partial_\mu \omega_\nu - \partial_\nu \omega_\mu \\ \vec{G}_{\mu\nu} &= \partial_\mu \hat{R}_\nu - \partial_\nu \hat{R}_\mu \end{aligned} \tag{2}$$

and ψ is an isospin doublet of protons and neutrons,

$$\psi = \begin{pmatrix} p \\ n \end{pmatrix} \tag{3}$$

In the above Lagrangian m_N, m_V, m_σ correspond to the nucleon, omega and rho meson and sigma mesons respectively, while g_S, g_V, and g_r are the corresponding coupling constants. The potential function $U(\sigma)$ is taken to be a general quartic polynomial of the form

$$U(\sigma) = \frac{m_S^2}{2} \sigma^2 + \frac{b}{3} \sigma^3 + \frac{c\sigma^4}{4} \quad . \tag{4}$$

Such a model of nuclear matter was first considered by Schiff,[5] who showed that it can lead to nuclear matter saturation. The properties of nuclear matter for a specific choice of b and c were considered by Boguta and Bodmer[6] and Boguta and Rafelski[7] (called the BB-model). The parameters were determined by fitting nuclear matter compressibility and surface energy together with binding energy, saturation density and symmetry energy. The choice of b = c leads to a model considered by Walecka.[8] Recently, the author has considered a model[9] (called the B-model) in which the values of b and c were fitted to reproduce the energy and density behavior of the single particle potential in nuclear matter predicted by the variational calculations of Pandharipande and Friedman.[10] This model describes closed shell nuclei very well.[11] In this work we investigate the effects of beta stability in the BB-model and the B-model. These models are of interest because both saturate symmetric nuclear matter at $\rho_0 = 0.1625/\text{fm}^3$ with binding energy of -15.75 MeV/particle and their low density equations of state are almost identical. The bulk properties of nuclear matter are equally well described by both models.

For infinite nuclear matter, assuming translational and rotational invariance for the ground state, the equations of motion become

$$m_s^2\sigma + b\sigma^2 + c\sigma^3 + g_s[\rho_s(p) + \rho_s(n)] = 0 \tag{5a}$$

$$m_v^2\omega_0 - g_v[\rho_v(p) + \rho_v(n)] = 0 \tag{5b}$$

$$m_v^2R_0^{(0)} - g_r[\rho_v(n) - \rho_v(p)] = 0 \tag{5c}$$

where ω_0 is the time component of the omega field and $R_0^{(0)}$ is the time component of the neutral rho field. The energy density ϵ is given by

$$\epsilon = \frac{1}{2}\left(\frac{g_v}{m_v}\right)^2 [\rho_v(n) + \rho_v(p)]^2 + \frac{1}{2}\left(\frac{g_r}{m_v}\right)^2 [\rho_v(n) - \rho_v(p)]^2 + U(\sigma)$$

$$+ 2\int^{k_F(n)} \frac{d^3k}{(2\pi)^3}\sqrt{k^2 + m^{*2}} + (n \longleftrightarrow p) \qquad (6)$$

$$m^* = m_N + g_s\sigma \qquad (7)$$

The pressure P at zero temperature is given by

$$P = \rho_B^2 \frac{d(\epsilon/\rho_B)}{d\rho_B} \qquad (8a)$$

with

$$\rho_B = \rho_v(n) + \rho_v(p) \qquad (8b)$$

The Fermi energies of the neutrons and protons are

$$E_F(n) = g_v\omega_o + g_r R_o^{(0)} + \sqrt{k_F(n)^2 + m^{*2}} \qquad (9a)$$

$$E_F(p) = g_v\omega_o - g_r R_o^{(0)} + \sqrt{k_F(p)^2 + m^{*2}} \qquad (9b)$$

The requirement of beta stability implies that the reaction $n \to p + e^- + \bar{\nu}_e$ has reached an equilibrium. That is

$$E_F(n) = E_F(p) + k_F(e) \qquad (10)$$

where we assume that electrons are relativistic ($k_F(e) \gg m_e$). Charge neutrality requires that

$$k_F(p) = k_F(e) \qquad (11)$$

Beta stability then becomes

$$2\left(\frac{g_r}{m_v}\right)^2 [\rho_v(n) - \rho_v(p)] + \sqrt{k\ (n)^2 + m^{*2}} - \sqrt{k\ (p)^2 + m^{*2}} = k_F(p) \qquad (12)$$

We have implicitly assumed that the neutrons and protons have the same effective mass m*. This means that we neglect the effects of the 0^{++} isovector field. This approximation is justified because the scalar meson nonet is taken to be $Q^2\bar{Q}^2$ quark configuration, and the isovector memeber has two strange quarks.[12]

For a nonrotating neutron star the equation of hydrostatic equilibrium is given by the Tolman-Oppenheimer-Volkov equations. They are

$$\frac{dP(r)}{dr} = - \frac{G[d(r) + P(r)/c^2]\,[m(r) + 4\pi r^3 P(r)/c^2]}{[r - 2Gm(r)/c^2]r} \tag{13a}$$

$$\frac{dm(r)}{d\,r} = 4\pi r^2 d(r) \tag{13b}$$

where P(r) is the pressure and m(r) is the amount of mass the spherically symmetric neutron star has up to a distance r away from the center of the star. Here d(r) is the energy-mass density. The relationship between the pressure and the particle density, or the energy-mass density, is given by the equation of state. Given a central energy-mass density d_c one computes the neutron star mass m(R) and the radius R by integrating the Tolman-Oppenheimer-Volkov equations to the point P(R) = 0.

The pressure P as a function of energy-matter density (d_c) is obtained by solving the field equations Eq. (5a-5c) together with the beta stability equation Eq. (12) for given values of $C_s = (g_s/m_s)/m_N$, $C_v = (g_v/m_v)m_N$, $C_r = (g_r/m_v)m_N$, $b/g_s^3 = \bar{b}$ and $c/g_s^4 = \bar{c}$. For the BB-model the values are $C_s = 8$, $C_v = 3$, $C_r = 5.8$, $\bar{b} = 1.76$, $\bar{c} = 6.82$. For the B-model they are $C_s = 15.6$, $C_v = 12.5$, $C_r = 4.4$, $\bar{b} = -5.2 \times 10^{-3}$, $\bar{c} = 2.46 \times 10^{-3}$. Both models saturate infinite symmetric nuclear matter at a density of $\rho = 0.1625/fm^3$ with a binding energy of -15.75 MeV/particle with a symmetry energy coefficient of 35 MeV. The BB-model has nuclear compressibility of $K = 200$ MeV and the B-model has $K = 280$ MeV. The two models derive nuclear saturation by different mechanisms. While in the BB-model the attraction and repulsion come mainly from the scalar field, that is, the nuclear force is highly non-Yukawa, in the B-model repulsion comes mainly from the ω-field with a small admixture of many body effects (σ^3 and σ^4 contributions). The implication of this for beta stability is dramatic.

In Fig. 1 we show the pressure as a function of baryon density for the BB-model and B-model for pure neutron matter [$\rho_V(p) = 0$] and for beta stable neutron matter. In the BB-model the difference in the equation of state for beta stable and pure neutron matter is very significant. For the B-model the difference in the equations of state is relatively small. The proton concentration in beta stable neutron matter at about $4\rho_0$ in both models is 30%.

The neutron star mass, as a function of energy-mass density is obtained by solving the Tolman-Oppenheimer-Volkov equations, Eq. (13a-13b), given the equation of state and the central density of the star. In Fig. 2 we show the results for B-model and BB-model for pure neutron matter and beta stable neutron matter. As expected, the mass of the star as a function of the central mass density in the B-model changes by about 20%, in beta stable stars. The central density at the maximum mass increases by about 10% since the equation of state is softer for beta stable neutron matter. The characteristics of neutron stars in the BB-model are considerably modified when the beta stability requirement is imposed.

The B-model and the BB-model were constructed to reproduce known bulk properties of symmetric nuclear matter at saturation. At, or below, nuclear saturation density, the two equations of state are almost identical. Thus we can conclude that the knowledge of bulk properties of nuclear matter at or below saturation is by far insufficient to determine the properties of beta stable neutron stars. Furthermore, the requirement of beta stability can rule out the BB-model as a reasonable model, which could not have been done just from the knowledge of the bulk properties of matter alone. The importance of beta stability cannot be prejudged beforehand but must be verified in each model to see whether it does not dramatically alter the predicted neutron star properties.

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Figure Captions

Fig. 1. Pressure as a function of baryon density in various relativistic field models.

Fig. 2. Neutron star mass as a function of mass density in various relativistic field models.

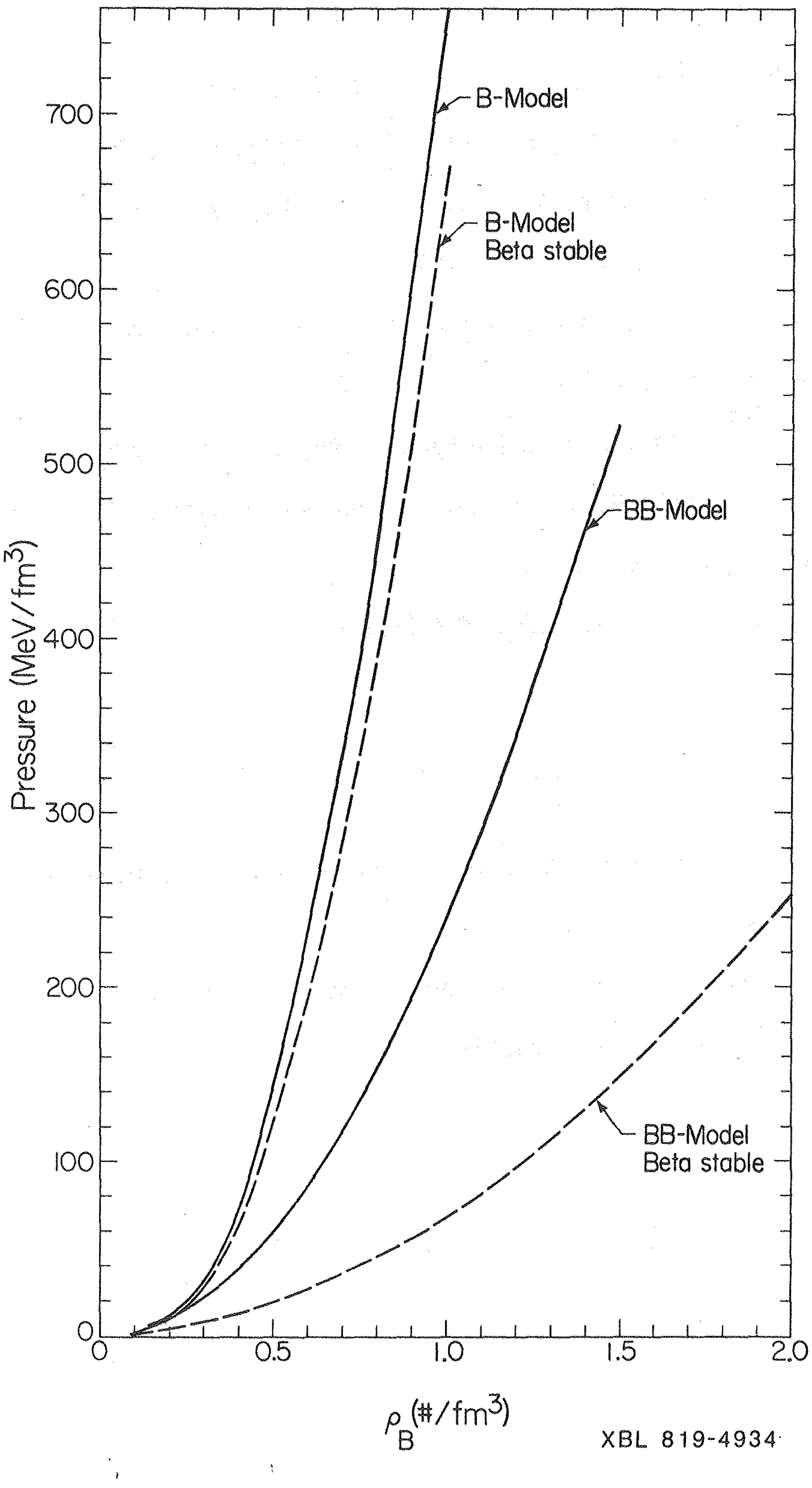
B-Model
B-Model
Beta stable
BB-Model
BB-Model
Beta stable
Pressure (MeV/fm³)
0
100
200
300
400
500
600
700
0
0.5
1.0
1.5
2.0
ρB(#/fm³)
XBL 819-4934

Fig. 1

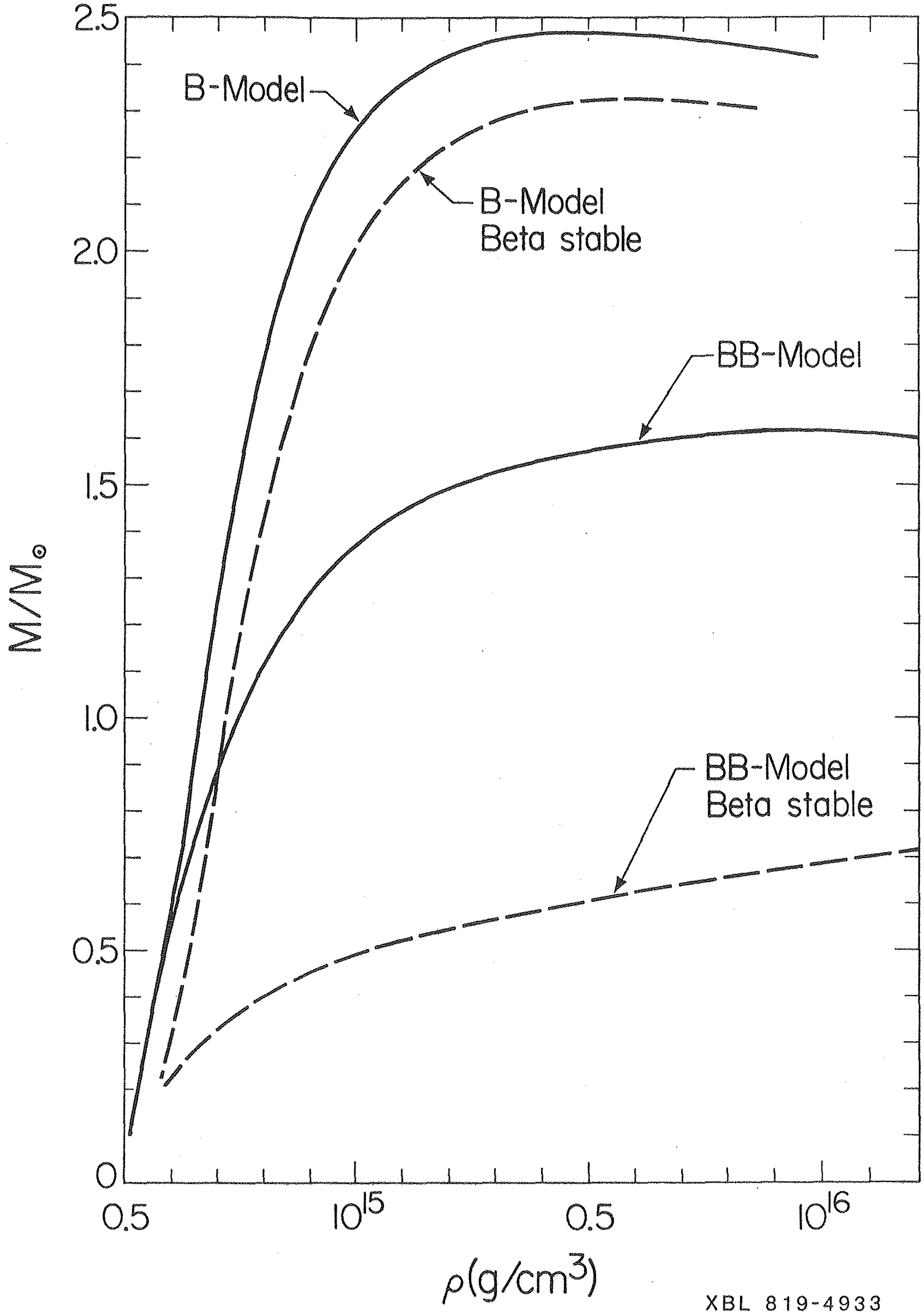

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Fig. 2